Vauxhall High School

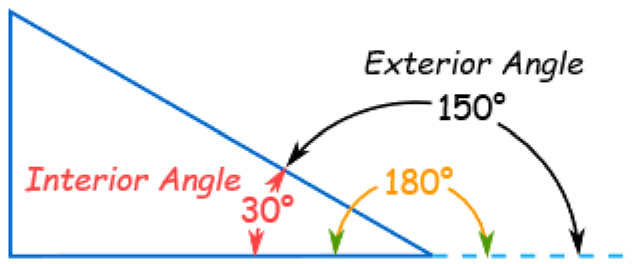
Mathematics

Grade 8

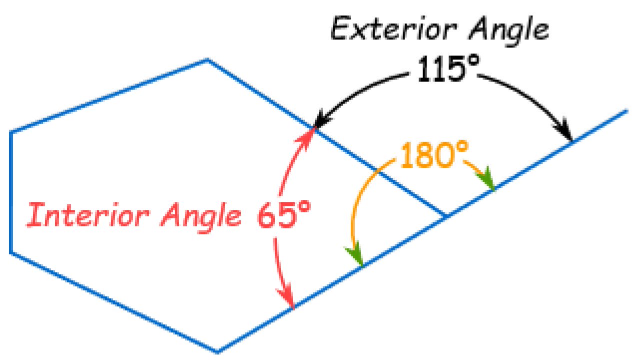
Notes March 30 – April 3

Interior Angles of Polygons

An Interior Angle is an angle inside a shape



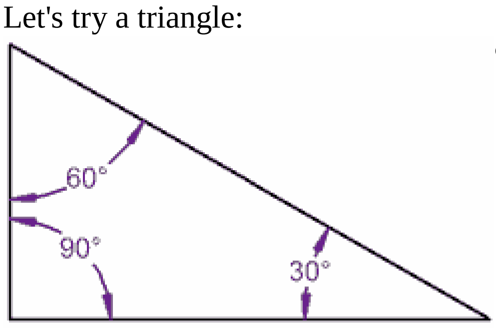
Another example:



Triangles

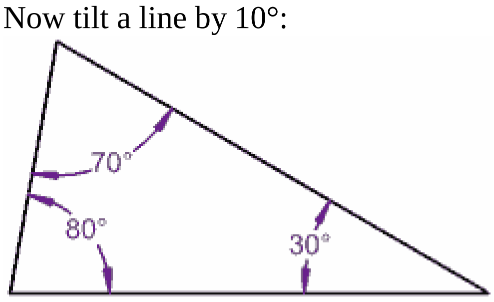
The Interior Angles of a Triangle add up to 180°

Let's try a triangle:



90° + 60° + 30° = 180° . It works for this triangle

Now tilt a line by 10°:



80° + 70° + 30° = 180°

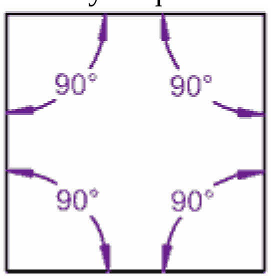
It still works!

One angle went **up** by 10°, and the other went **down** by 10°

Quadrilaterals (Squares, etc)

(A Quadrilateral has 4 straight sides)

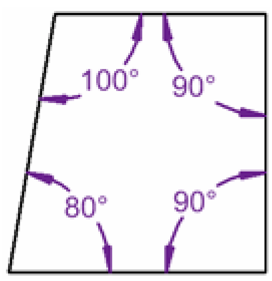
Let's try a square:



90° + 90° + 90° + 90° = 360°

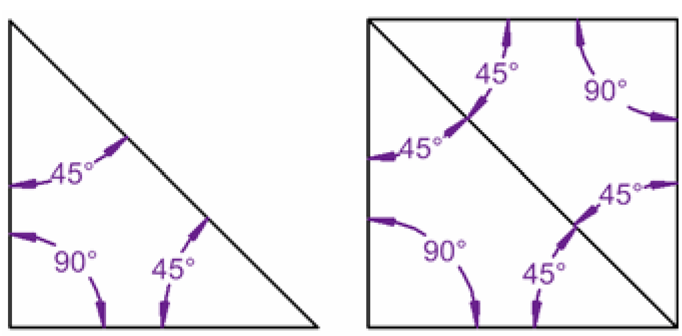
A Square adds up to 360°

Now tilt a line by 10°:



80° + 100° + 90° + 90° = 360°. It still adds up to 360°. The Interior Angles of a Quadrilateral add up to 360°

Because there are 2 triangles in a square ...

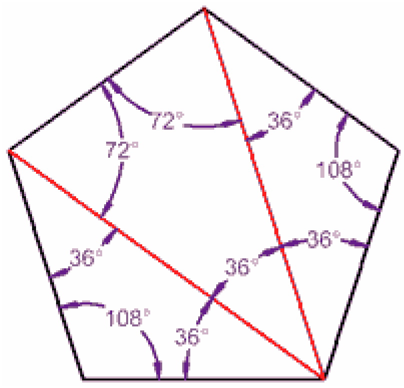


The interior angles in a triangle add up to **180°** ...

... and for the square they add up to **360°** ...

... because the square can be made from two triangles!

Pentagon



A pentagon has 5 sides, and can be made from **three triangles**, so you know what ...

... its interior angles add up to 3 × 180° = **540°**

And when it is **regular** (all angles the same), then each angle is 540° / 5 = 108°

(Exercise: make sure each triangle here adds up to 180°, and check that the pentagon's interior angles add up to 540°)

The Interior Angles of a Pentagon add up to 540°

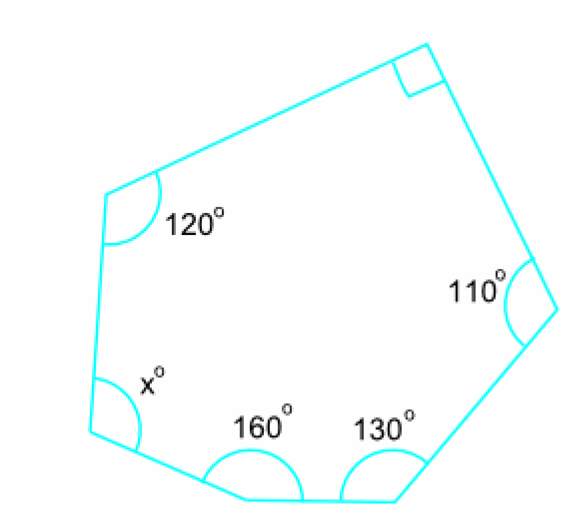
The General Rule

Each time we add a side (triangle to quadrilateral, quadrilateral to pentagon, etc), we **add another 180°** to the total:

Formula

Sum of Interior Angles = (**n**-2) × 180°

Example 1:



The diagram shows a hexagon. What is the size of the angle x°?

Use the formula:

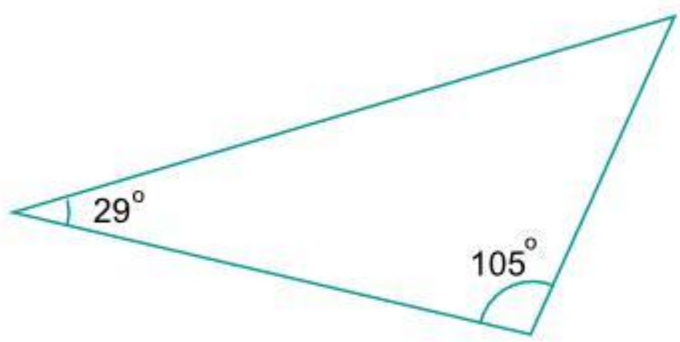
Sum of interior angles = (n − 2) × 180°
with n = 6 (because a hexagon has 6 sides)

⇒ The interior angles of a hexagon add to 4 × 180° = 720°

The sum of the given angles = 120° + 90° + 110° + 130° + 160° = 610°

So the sixth angle = 720° − 610° = 110°
So x = 110°

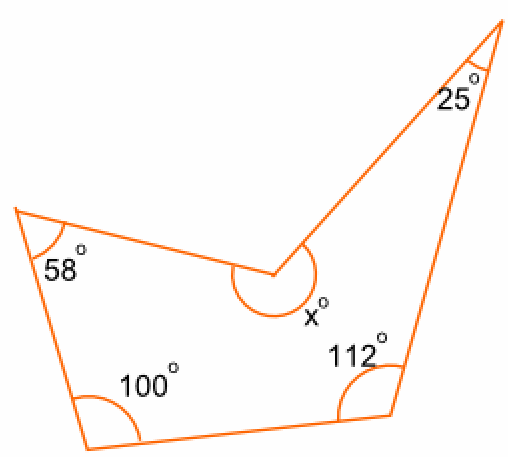
Example 2:
What is the third interior angle of the triangle?



The interior angles of a triangle add to 180°
The sum of the given angles = 29° + 105° = 134°
Therefore the third angle = 180° − 134° = 46°

Example 3:

The diagram shows a pentagon. What is the size of the angle x°?



Use the formula:
Sum of interior angles = (n − 2) × 180°
with n = 5 (because a pentagon has 5 sides)

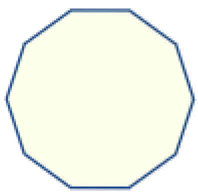
⇒ The interior angles of a pentagon add to 3 × 180° = 540°

The sum of the given angles = 58° + 100° + 112° + 25° = 295°
So the fifth angle = 540° − 295° = 245°

Note that this is a reflex angle (an angle greater than 180°, but less than 360°) and is the correct answer for the interior angle marked.

So x = 245°

Example4: What about a Regular Decagon (10 sides) ?



Sum of Interior Angles = (**n**-2) × 180°
= (**10**-2)×180° **=** 8×180° = **1440°**

And it is a Regular Decagon so:

Each interior angle = 1440°/10 = **144°**

Example 5: Each of the interior angles of a regular polygon is 150°. How many sides does the polygon have?

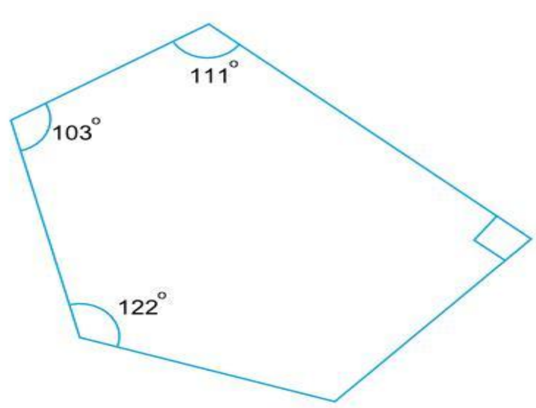
Use the formula for one angle of a regular n-sided polygon:

We know one angle = 150°

Multiply throughout by n:

Therefore (n − 2) × 180 = 150n
⇒ 180n − 360 = 150n
⇒ 180n − 150n = 360
⇒ 30n = 360
⇒ n = 360 ÷ 30 = 12

Example 6 : What is the fifth interior angle of the pentagon?



The interior angles of a pentagon add to 540°.

The sum of the given angles = 111° + 103° + 122° + 90° = 426°
(The angle marked with a square is a right angle, which is 90°)

So the fifth angle = 540° − 426° = 114°

Example 7: What is the size of one interior angle of a regular nonagon (nine-sided polygon)?

Use the formula for one angle of a regular n-sided polygon:

$$\frac{(n-2)\times 180^\circ}{n} \text{ with } n = 9$$

$$= \frac{7 \times 180^\circ}{9} = 140^\circ$$

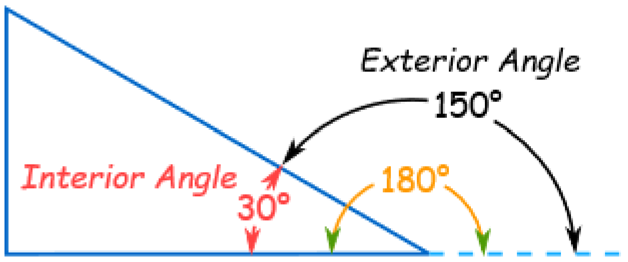
Example 8: What is the sum of the interior angles of a regular dodecagon (12-sided polygon)?

Use the formula:
Sum of interior angles = (n - 2) × 180°
with n = 12

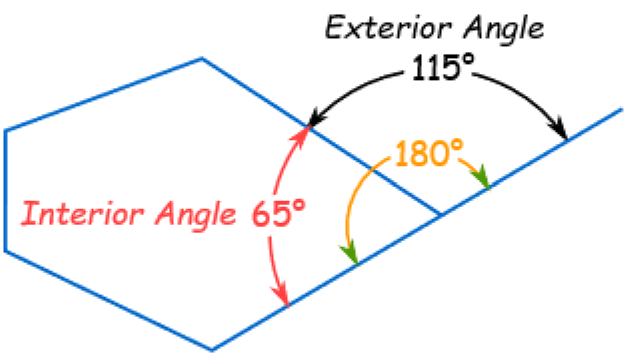
Sum of interior angles of a regular dodecagon = (12 - 2) × 180° = 10 × 180° = 1,800°

Exterior Angle

The Exterior Angle is the angle between any side of a shape, and a line extended from the next side.



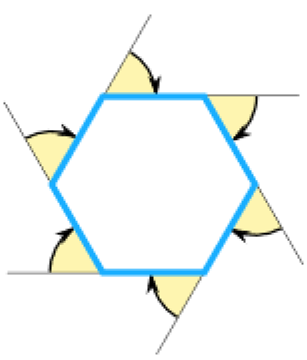
Another example:



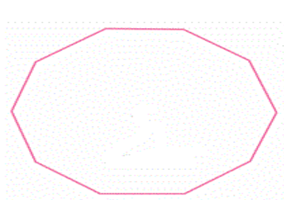
When we add up the Interior Angle and Exterior Angle we get a straight line 180°. They are Supplementary Angles.

The Exterior Angles of a Polygon add up to 360°. In other words the exterior angles add up to **one full revolution**. This rule only works for simple polygons.

Each lines changes direction until we eventually get back to the start:



Example 1: What is the size of one exterior angle of a regular decagon (ten-sided polygon)?



The exterior angles of any polygon add up to 360°
So the exterior angles of a regular decagon must also add up to 360°
Therefore, one exterior angle of a regular decagon = 360° ÷ 10 = 36°

Example 2: One exterior angle of a regular polygon is 20°. How many sides does it have?

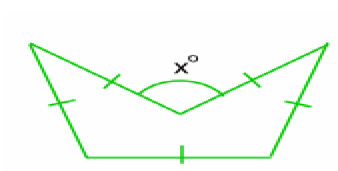
The exterior angles of a polygon add up to 360°

So if the polygon is regular and has n sides, one exterior angle must be: 360°/n

$360°/n = 20°$
$\Rightarrow 20 \times n = 360$
$\Rightarrow n = 360 \div 20 = 18$

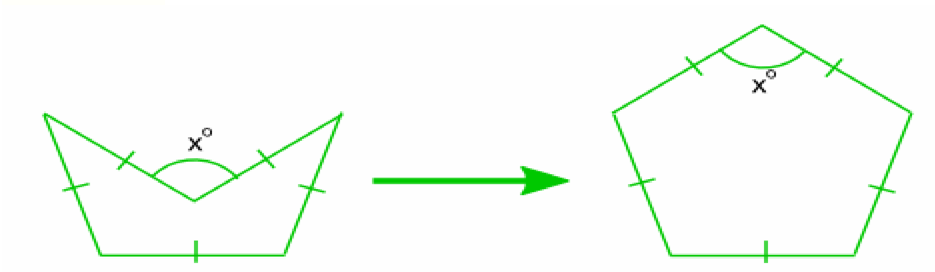
The polygon has 18 sides.

Example 3: The diagram shows a polygon with five equal sides. What is the size of angle x°?



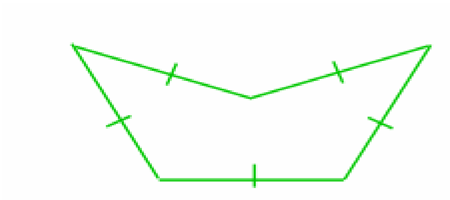
This is a trick question because the polygon is not regular.
A regular polygon has all its sides equal in length **AND** all its angles equal.

VISUALLY, by rearranging the sides of the pentagon to make a regular pentagon, as follows:



we can guess that the angle marked x° is equal to one interior angle of a regular pentagon
= 540° ÷ 5 = 108°

But in fact the angles can be changed (while keeping the sides the same length) like this:



so we can't really be sure what the angle is.

Example 4: The exterior angles of a heptagon are y°, 2y°, 3y°, 3y°, 4y°, 5y° and 6y°. What is the value of y?

For any polygon, the sum of the exterior angles = 360°

Therefore $y° + 2y° + 3y° + 3y° + 4y° + 5y° + 6y° = 360°$
$\Rightarrow 24y = 360 \Rightarrow y = 360 \div 24 = 15$

Example 5:

The exterior angles of an octagon are x°, 2x°, 3x°, 4x°, 5x°, 6x°, 7x°, and 8x°
What is the size of the smallest interior angle of this octagon?

For any polygon, the sum of the exterior angles = 360°

Therefore $x° + 2x° + 3x° + 4x° + 5x° + 6x° + 7x° + 8x° = 360°$
$\Rightarrow 36x = 360 \Rightarrow x = 360 \div 36 = 10$

Therefore the size of the largest exterior angle = $8 \times 10° = 80°$
$\Rightarrow$ the size of the smallest interior angle = $180° - 80° = 100°$