VAUXHALL HIGH SCHOOL

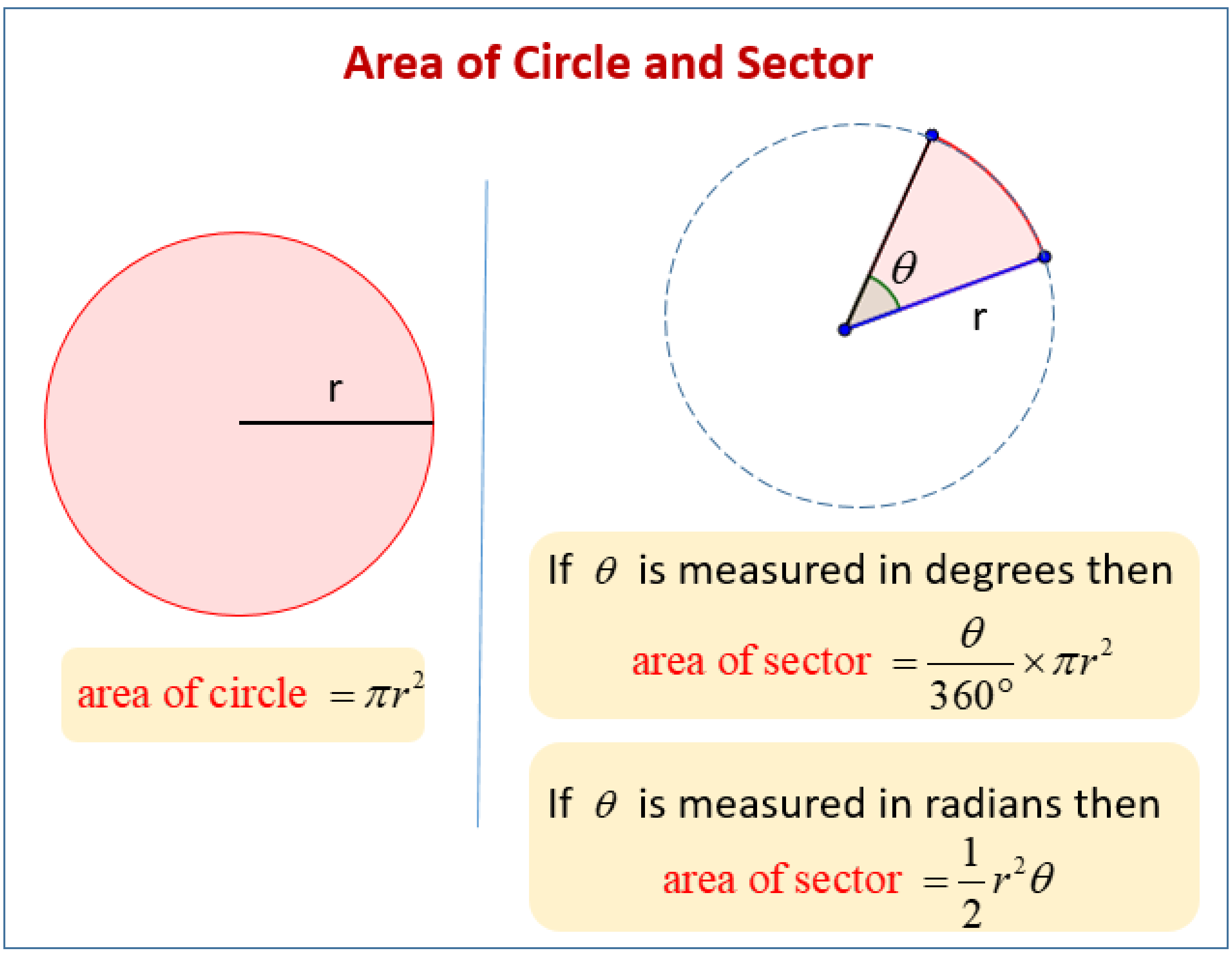
Grade : 9
TOPIC : Geometry
SUB – TOPIC : Area of Sector and Segment, Length of arc

Area of Sector

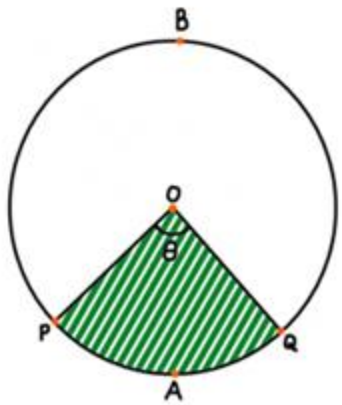
A sector is like a "pizza slice" of the circle. It consists of a region bounded by two radii and an arc lying between the radii.

The area of a sector is a fraction of the area of the circle. This area is proportional to the central angle. In other words, the bigger the central angle, the larger is the area of the sector.

The following diagrams give the formulas for the area of circle and the area of sector. Scroll down the page for more examples and solutions.



In the figure below, *OPBQ* is known as the **Major Sector** and *OPAQ* is known as the **Minor Sector.** As Major represent big or large and Minor represent Small, which is why they are known as Major and Minor Sector respectively. In a semi-circle, there is no major or minor sector.



Formula for Area of Sector (in degrees)

We will now look at the formula for the area of a sector where the central angle is measured in degrees.

Recall that the angle of a full circle is 360˚ and that the formula for the area of a circle is πr^2.

Comparing the area of sector and area of circle, we derive the formula for the area of sector when the central angle is given in degrees.

$$\frac{\text{Area of Sector}}{\text{Area of Circle}} = \frac{\text{Central Angle}}{360^\circ}$$

$$\frac{\text{Area of Sector}}{\pi r^2} = \frac{\text{Central Angle}}{360^\circ}$$

$$\text{Area of Sector} = \frac{\text{Central Angle}}{360^\circ} \times \pi r^2$$

where *r* is the radius of the circle

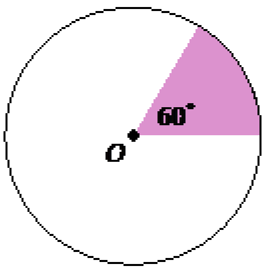
NB: Focus on the formula that uses degrees instead of radians

Calculate The Area Of A Sector (using formula in degrees)

We can calculate the area of the sector, given the central angle and radius of circle.

Example:

Given that the radius of the circle is 5 cm, calculate the area of the shaded sector. (Take π = 3.142).



Solution:

$$\text{Area of Sector} = \frac{\text{Central Angle}}{360°} \times \pi r^2$$

Area of sector = $\frac{60°}{360°} \times 25\pi$

$=13.09\ cm^2$

Calculate Central Angle Of A Sector

We can calculate the central angle subtended by a sector, given the area of the sector and area of circle.

Example:

The area of a sector with a radius of 6 cm is 35.4 cm^2. Calculate the angle of the sector. (Take π = 3.142).

Solution:

$$\text{Area of Sector} = \frac{\text{Central Angle}}{360°} \times \pi r^2$$

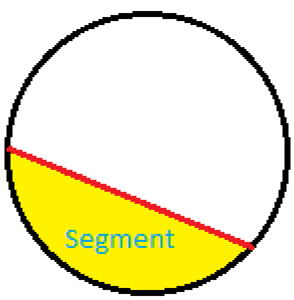
Central Angle = $\frac{35.4}{36\pi} \times 360°$

= 112.67°

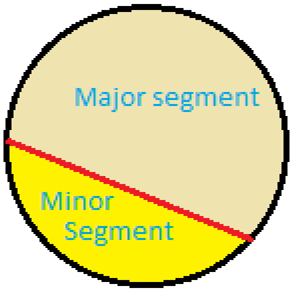
Area of Segment (angle in degrees)

A **segment** is the section of a circle enclosed by a chord and an arc. Therefore, those halves of the pizza are segments. If you eat one half, you would have eaten a semicircle (half of a circle), which is the biggest segment of a circle.

Since a circle has an infinite number of points on the circumference, there are many possibilities for a chord and, hence, many possibilities for segments.

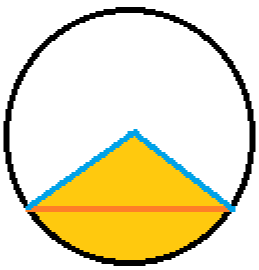


When a circle is divided into two segments of different areas, the biggest segment is called the major segment and the smaller segment is called the minor segment.



Now, if you order pizza, how does it usually come? Yes, pre-sliced. Those slices look like triangles with a curve, right? Those are sectors. A **sector** of a circle is the section enclosed by two radii (the two sides of the slice) and an arc (the crust).

We can divide a sector so that it looks like a triangle and a segment.

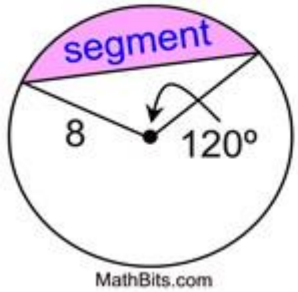


We divided the sector so that a triangle is formed and a segment is formed. If we were to consider the total area of that sector, we could say that the total area of the sector is made up of the area of the triangle and the area of the segment. Some people don't like the crust of the pizza. So when they get a slice, they cut off the crust. However, their full slice includes the crust, plus the part that is left. Therefore, we can say that a sector is made up of the triangle and the segment (the crust).

Using that example, we can find the area of a segment using the formula:

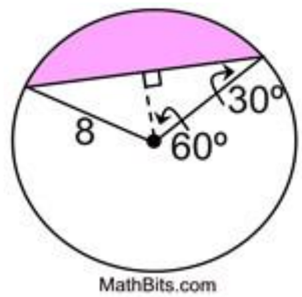
Area of Segment = Area of Sector - Area of Triangle

Example: Find the area of a segment of a circle with a central angle of 120 degrees and a radius of 8 cm. Express answer to the ***nearest integer.***



Solution: Start by finding the area of the sector.

$$A = \frac{120}{360}\pi(8)^2 = \frac{1}{3}\pi(64) = \frac{64}{3}\pi \approx 67.0206432$$



Now, find the area of the triangle. Dropping the altitude from the center forms a 30-60-90 degree triangle. Using the 30-60-90 rules (or trigonometry), find the altitude, which is 4, and the other leg, which is $4\sqrt{3}$ or 6.92820323.

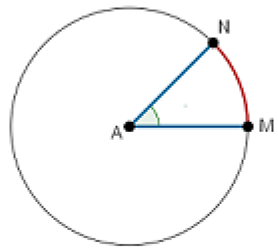
$$A_{\Delta} = \frac{1}{2}bh = \frac{1}{2}(2 \times 6.92820323)(4) = 27.71281292$$

$$A_{segment} = A_{sec\,tor} - A_{triangle}$$
$$A_{segment} = 67.02064328 - 27.71281292$$
$$A_{segment} = 39.30783036 = 39 \text{ sq.cm.}$$

Video link on Area of segment calculations: https://youtu.be/mjFwYFjd7Q4

Arc of a Circle

An **arc** is any connected part of the circumference of a circle.



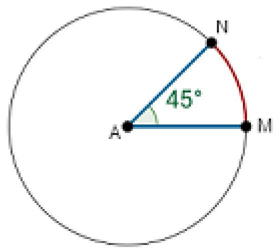
In the diagram above, the part of the circle from M to N forms an arc. It is called arc MN.

An arc could be a minor arc, a semicircle or a major arc.

- A semicircle is an arc that is half a circle.
- A minor arc is an arc that is smaller than a semicircle.
- A major arc is an arc that is larger than a semicircle.

Central Angle

A central angle is an angle whose vertex is at the center of a circle.



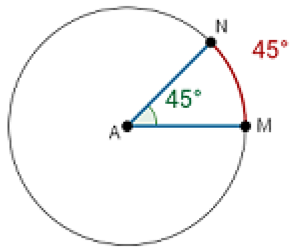
In the diagram above, the central angle for arc MN is 45°.

The sum of the central angles in any circle is 360°.

Arc Measure

The measure of a semicircle is 180°.

The measure of a minor arc is equal to the measure of the central angle that intercepts the arc. We can also say that the measure of a minor arc is equal to the measure of the central angle that is subtended by the arc. In the diagram below, the measure of arc MN is 45°.



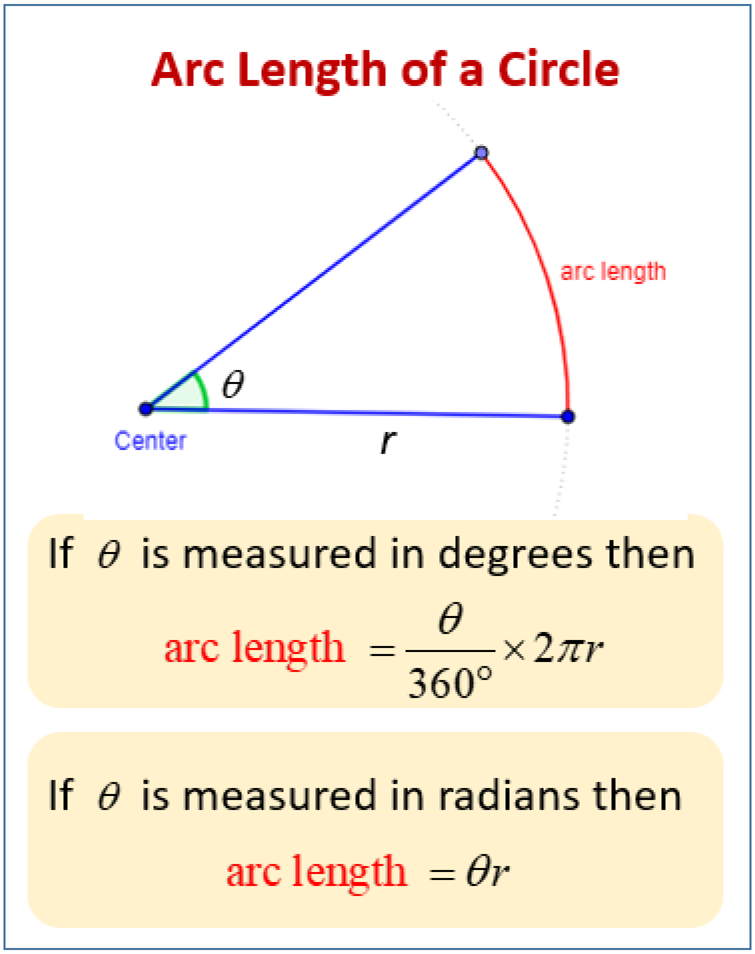
The measure of the major arc is equal to 360° minus the measure of the associated minor arc.

Video link on minor and major arcs: https://youtu.be/L4l05-ydHBs

Arc Length Formula

The arc length is the distance along the part of the circumference that makes up the arc.

The following diagram gives the formulas to calculate the arc length of a circle for angle measures in degrees and angle measured in radians. Scroll down the page for more examples and solutions.



Arc Measure given in Degrees

Since the arc length is a fraction of the circumference of the circle, we can calculated it in the following way. Find the circumference of the circle and then multiply by the measure of the arc divided by 360°. Remember that the measure of the arc is equal to the measure of the central angle.

The formula for the arc length of a circle is

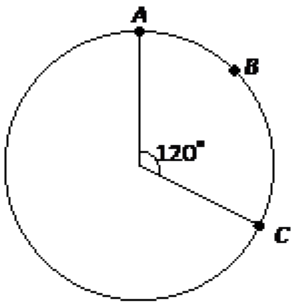
$$\text{Arc Length} = 2\pi r \times \frac{m}{360^\circ}$$

where *r* is the radius of the circle and *m* is the measure of the arc (or central angle) in degrees.

Calculate Arc Length given Measure of Arc in degrees

From the formula, we can calculate the length of the arc.

Example:



If the circumference of the following circle is 54 cm, what is the length of the arc *ABC*?

Solution:

Circumference = 2πr = 54 cm

$$\begin{aligned}\text{Arc Length} &= 2\pi r \times \frac{m}{360^\circ}\\ &= 54 \times \frac{120^\circ}{360^\circ}\\ &= 18cm\end{aligned}$$

Example:

If the radius of a circle is 5 cm and the measure of the arc is 110˚, what is the length of the arc?

Solution:

$$\begin{aligned}\text{Arc Length} &= 2\pi r \times \frac{m}{360^\circ}\\ &= 2\pi \times 5 \times \frac{110^\circ}{360^\circ}\\ &= 9.6cm\end{aligned}$$

Video Link on how to calculate the arc length given the central angle: https://youtu.be/anV3HI-1vyY